GENERAL SOLUTIONS OF TRIGONOMETRIC EQUATIONS

A) Particular solutions to Trig Equations (solving without graphing in GC)

We solve Trig equations like one would algebraic equations. This approach will lead us to one solution. This solution is generally the principal solution i.e. the angle that is closest to 0°.

Example 1: Solve $3\cos x + 2 = 1$.

Method:
$$3\cos x + 2 = 1$$
$$3\cos x = -1$$
$$\cos x = \frac{-1}{3}$$
$$x = \cos^{-1}\left(\frac{-1}{3}\right)$$
$x = 1.91$ rad or $109.47°$.

Example 2: Solve $\sin(2x - \frac{\pi}{3}) = 1$.

Method:
$$\sin(2x - \frac{\pi}{3}) = 1$$
$$\left(2x - \frac{\pi}{3}\right) = \sin^{-1}(1)$$
you need to be in radians, since the angle contains $\frac{\pi}{3}$
$\left(2x - \frac{\pi}{3}\right) = 1.57$ (or $\frac{\pi}{2}$) rad
$$2x = \frac{\pi}{2} + \frac{\pi}{3}$$
$$x = \frac{5\pi}{6} \div 2$$
$x = \frac{5\pi}{12}$ or 1.31 rad

B) General Solutions of Trig Equations

GENERAL SOLUTIONS FORMULAE

If $\sin\theta = \sin\alpha$, then $\theta = n\pi + (-1)^n \cdot \alpha$

If $\cos\theta = \cos\alpha$, then $\theta = 2n\pi \pm \alpha$

If $\tan\theta = \tan\alpha$, then $\theta = n\pi + \alpha$

where n is any integer.

Example 1: Find the general solution of the equation $3\cos x = -2$.

Method: $3\cos x = -2$

$\cos x = \frac{-2}{3}$

$x = \cos^{-1}(\frac{-2}{3})$

$x = 2.3005$ rad *this is your principal soln, your value for α*

$x = 2n\pi \pm 2.3005$ *this is your General Solution.*

The general solution allows you to find the multiple values of x, i.e. the multiple solutions to the equation $3\ \cos x = -2$.

If you wanted to find solutions within the range $0 \leq x \leq 2\pi$, you would sub in $n = 0, n = 1, n = 2$ etc until you found all the solutions within that range.

If you wanted to find solutions within the range $-\pi \leq x \leq \pi$, you would sub in $n = 0, n = 1, n = -1$ i.e. include negative values for n, until you found all the solutions within that range.

Example 2: Find the general solution of the equation $\sin(3x + \pi) = -0.468$ and then find all the solutions between $x = 0$ and $x = 2\pi$.

- Method:

$\sin(3x + \pi) = -0.468$

$(3x + \pi) = \sin^{-1}(-0.468)$

$(3x + \pi) = -0.48703$ rad *this is your principal soln, your value for α*

$(3x + \pi) = n\pi + (-1)^n \cdot (-0.48703)$ *this is your General Solution*

- Now we can simplify the General Solution:

$(3x + \pi) = n\pi + (-1)^n \cdot (-0.48703)$

$3x = n\pi + (-1)^n \cdot (-0.48703) - \pi$

$x = \frac{1}{3} \cdot [n\pi + (-1)^n \cdot (-0.48703) - \pi]$

- Now we will find all the solutions within the range $0 \leq x \leq 2\pi$:

$n = 0: x = \frac{1}{3} \cdot [0\pi + (-1)^0 \cdot (-0.48703) - \pi] = \frac{1}{3}[0 + 1 \cdot (-0.48703) - \pi] = -1.2095$ not in range

$n = 1: x = \frac{1}{3} \cdot [1\pi + (-1)^1 \cdot (-0.48703) - \pi] = \frac{1}{3}[\pi + (-1) \cdot (-0.48703) - \pi] = 0.16234$ in range

$n = 2: x = \frac{1}{3} \cdot [2\pi + (-1)^2 \cdot (-0.48703) - \pi] = \frac{1}{3}[2\pi + 1 \cdot (-0.48703) - \pi] = 0.8849$ in range

$n = 3: x = \frac{1}{3} \cdot [3\pi + (-1)^3 \cdot (-0.48703) - \pi] = \frac{1}{3}[3\pi + (-1) \cdot (-0.48703) - \pi] = 2.2567$ in range

$n = 4: x = \frac{1}{3} \cdot [4\pi + (-1)^4 \cdot (-0.48703) - \pi] = \frac{1}{3}[4\pi + 1 \cdot (-0.48703) - \pi] = 2.9793$ in range

$n = 5: x = \frac{1}{3} \cdot [5\pi + (-1)^5 \cdot (-0.48703) - \pi] = \frac{1}{3}[5\pi + (-1) \cdot (-0.48703) - \pi] = 4.3511$ in range

$n = 6: x = \frac{1}{3} \cdot [6\pi + (-1)^6 \cdot (-0.48703) - \pi] = \frac{1}{3}[6\pi + 1 \cdot (-0.48703) - \pi] = 5.0736$ in range

So the solutions are: $x = 0.162, 0.885, 2.257, 2.979, 4.351\ and\ 5.074$ radians.